Residual Risk: How Much is Too Much?

Artificial limits on a portfolio's residual risk can lead to suboptimal behavior on the part of investors and managers.

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In portfolio management, excess return measures the difference between the portfolio's returns and those of an underlying benchmark, and residual risk measures the volatility of those excess returns. An investor who is averse to incurring residual risk relative to a benchmark is not risk-averse in the same sense that an investor who shies away from stocks in favor of bonds and cash is risk-averse. The latter investor does not want to incur the riskiness associated with stock returns. The former investor may be willing to incur the risk associated with stocks, or at least those stocks represented by the benchmark, but is more or less averse to incurring the additional risks that are associated with security selection.

For such an investor, gains and losses that come from holding the benchmark are not a matter for concern, but gains and losses relative to the benchmark are of some importance. As Clarke, Krase, and Statman put it [1994, p. 19]:

> Gains and losses that come with holding the benchmark portfolio are an "act of God." Gains and losses that come with deviation from the benchmark portfolio are an "act of man." Choice involves responsibility, and responsibility brings the pain of regret when the choice turns out badly.

This investor is regret-averse, rather than risk-averse in the traditional sense.

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Clarke et al. explain the difference by an analogy to a lottery participant who has bet on the same numbers for some time but is now considering a new set of numbers. The odds of either set of numbers winning are the same. But the lottery participant would feel extreme regret if he were to change numbers and the old numbers won; choice of a new set of numbers entails a high risk of regret.

Some investors, like lottery participants, wish to hold only the number represented by a given equity index. These investors are willing to accept the risks associated with holding the equity index, in exchange for receiving equity returns, but are so regret-averse that they are unwilling to incur any additional risks. These investors are likely to hold passive, indexed portfolios.

Other investors, however, may be willing to incur the residual risks associated with active security selection in exchange for expected excess returns. How much residual risk should they incur? The answer will depend upon the investor's aversion to residual risk and the portfolio manager's skill. The values of these parameters can be estimated, but the task is frequently simplified by placing simple constraints on portfolio residual risk levels.

Consultants and managers often categorize portfolios into specific ranges of residual risk. For example, "enhanced passive" or "index-plus" portfolios (which account for about $100 billion of institutional assets) are typically bounded by residual risks relative to a benchmark of between 0.30% and 2.00% (Schramm [1995, p. 3]). Their expected excess returns are generally between 0.15% and 1.00%. The next tier of residual risk portfolios, those having residual risk levels over 2%, are classified as "core" strategies.

A constraint such as a 2% limit on residual risk in effect brings down a curtain, beyond which lie excess returns and residual risks unavailable to the investor. Does this make sense? Shouldn't the investor be aware of what lies beyond the curtain, if only to understand what is being given up?

BEYOND THE CURTAIN

A framework developed by Grinold and Kahn [1995, pp. 91-99] can provide some guidance for understanding what lies beyond the 2% curtain. Development of a complete picture depends crucially upon the notion of the information ratio as a measure of the portfolio manager's skill. The information ratio, IR, is the maximum ratio of annualized excess return, α, to annualized residual risk, ω, the manager can obtain:[1]

$$IR = \alpha/\omega \tag{1}$$

The IR is assumed to be constant over all risk levels (i.e., excess return will increase proportionally with residual risk).[2] A good manager might have an IR of 0.5, while an exceptional manager might have an IR of 1.0.

For any given level of residual risk, ω, the objective is to maximize investor utility, U, defined as portfolio excess return less the disutility of portfolio residual risk:

$$U = \alpha - (\lambda \times \omega^2) \tag{2}$$

Investor utility increases with increases in portfolio excess return. Increases in portfolio residual risk, however, reduce investor utility by a factor, λ, that reflects the investor's aversion to residual risk (regret aversion).

Substituting from Equation (1), investor utility can be expressed in terms of residual risk, investor regret aversion, and manager IR:

$$U = (\omega \times IR) - (\lambda \times \omega^2) \tag{3}$$

Utility will increase with an increase in IR and decrease with increases in the investor's level of residual risk aversion. The optimal level of aggressiveness or residual risk, ω^*, for a portfolio will also increase with IR and decrease with aversion to residual risk:[3]

$$\omega^* = IR/2\lambda \tag{4}$$

Exhibit 1 illustrates some of the trade-offs involving residual risk, excess return, investor aversion to risk, and manager skill. The two lines ascending from the zero-residual risk, zero-excess return origin (the underlying benchmark) represent various possible combinations of excess return and residual risk that could be offered by two managers.[4] The first manager has an IR of 1.0; the portfolios on this frontier offer excess returns equal to their residual risks. The second manager has an IR of 0.5; the portfolios on this frontier offer excess returns half the magnitude of their residual risks.

The points H, M, and L on the efficient frontiers illustrate the optimal portfolios for investors with three levels of aversion to residual risk — 0.15 (high), 0.10

EXHIBIT 1
INVESTOR RISK AVERSION AND MANAGER SKILL

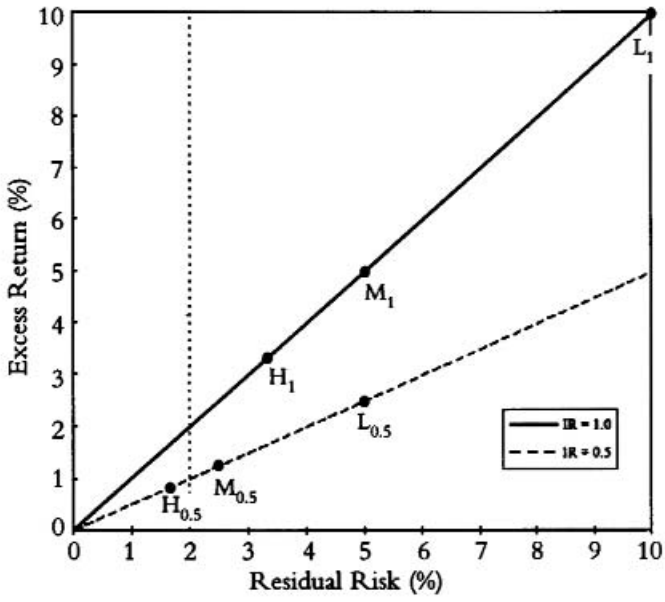
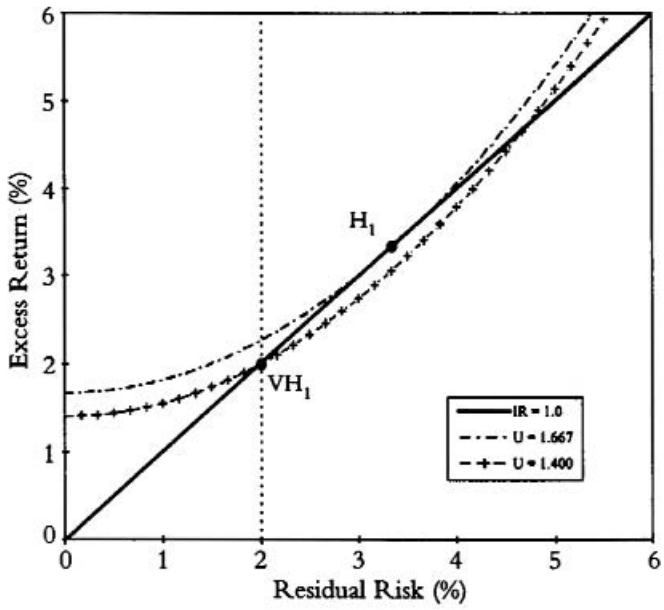


EXHIBIT 2
SACRIFICE IN UTILITY FROM OVERESTIMATING INVESTOR RISK AVERSION



(medium), and 0.05 (low). We can place some numbers on these points, using Equation (4). Given a manager with an IR of 1.0, the optimal portfolios for investors with high, medium, and low aversions to residual risk, H_1, M_1, and L_1, will have residual risk levels of 3.33%, 5.00%, and 10.00%, respectively. Given a manager with an IR of 0.5, the optimal portfolios, $H_{0.5}$, $M_{0.5}$, and $L_{0.5}$, will have residual risk levels of, respectively, 1.67%, 2.50%, and 5.00%.

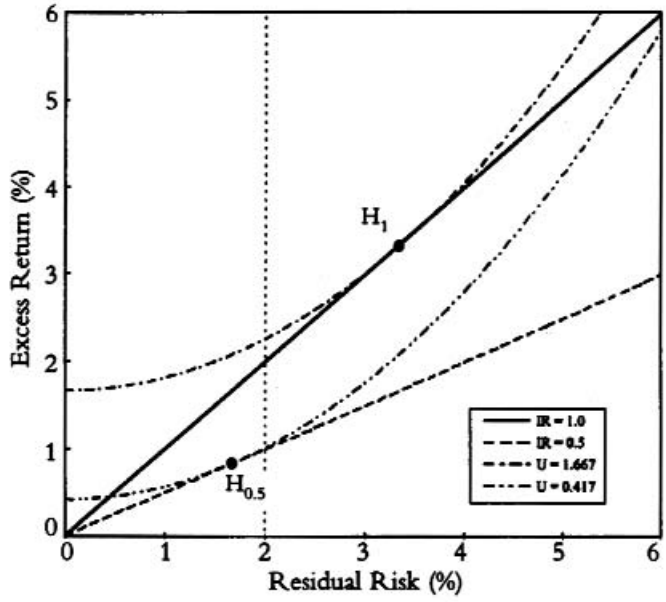
Note that, along both frontiers, higher levels of residual risk are associated with higher expected excess returns. Furthermore, the optimal (for the assumed risk tolerances) portfolios of the higher-IR manager have both higher residual risks and higher expected excess returns than those of the lower-IR manager. Higher expected excess returns accrue to higher-residual risk portfolios and to higher-IR managers.

The dotted vertical line in Exhibit 1 represents a 2% residual risk cutoff. Note that only one portfolio falls within this boundary — the portfolio corresponding to the high-regret aversion investor with the IR = 0.5 manager. The medium- and low-regret aversion portfolios on the IR = 0.5 frontier and all three portfolios on the IR = 1.0 frontier have residual risks above 2%. These portfolios would be unavailable to the investor with a 2% residual risk constraint.

In Exhibit 2, point VH_1 on the IR = 1.0 frontier represents a portfolio with a residual risk level of 2%. According to Equation (4), this portfolio will be optimal for an investor with a regret aversion level of 0.25 — a very high level of aversion to residual risk. Point H_1 represents the optimal portfolio for the investor with a high regret aversion level of 0.15 and a manager with an IR of 1.0. This portfolio is located at the point of tangency between the IR = 1.0 manager's efficient frontier and the utility curve for an investor with a regret aversion level of 0.15. All points on this curve are equally desirable for an investor with this level of regret aversion. The investor is thus indifferent between portfolio H_1 and a certain excess return of 1.667% (the certainty-equivalent found at the curve's intersection with the vertical axis).

The investor with residual risk aversion of 0.15 who opts for portfolio VH_1 because of a 2% constraint on residual risk will suffer a loss in utility. This loss can be calculated, using Equation (3), as the difference between the utility of portfolio H_1 (1.667%) and the utility of portfolio VH_1 (1.400%), assuming the investor's residual risk aversion is actually 0.15 and the manager's IR is 1.0. The magnitude of this sacrifice — 0.267 percentage points — is the distance between the utility curve passing through point H_1 and the curve

EXHIBIT 3
SACRIFICE IN UTILITY FROM USING LESS SKILLFUL MANAGER



passing through point VH_1. It corresponds to a certainty-equivalent sacrifice of 0.267 percentage points.

As noted above, the efficient frontier for an IR = 0.5 manager will be lower than that of an IR = 1.0 manager. Point $H_{0.5}$ in Exhibit 3 represents the optimal portfolio for an investor with regret aversion of 0.15 and a manager with an IR of 0.5. Point $H_{0.5}$, with residual risk of 1.67%, is well within the 2% curtain. It offers the best deal for the investor if there is no manager with a higher IR, who can offer more return at the investor's regret tolerance level. An investor who settles for portfolio $H_{0.5}$ when portfolio H_1 is available, however, will sacrifice 1.25 percentage points (1.667 – 0.417) in utility.

Of course, investing in portfolio H_1 means accepting a residual risk level above 2%. Does this imply that the investor constrained to a residual risk level of 2% or less should stick with portfolio $H_{0.5}$, even if a superior manager can be found? A better solution for the investor would be to dilute the residual risk of portfolio H_1 by investing some portion of funds in the underlying benchmark index.

Exhibit 4 shows that portfolio IH_1, evenly divided between a passive indexed portfolio and portfolio H_1 and having half the residual risk of portfolio H_1 alone, will lie directly above portfolio $H_{0.5}$ at the same risk level. Portfolio IH_1 is stochastically dominant to portfolio $H_{0.5}$; it offers higher expected excess return at the same level of residual risk. Its utility will be 1.250%. Compared with portfolio $H_{0.5}$, with utility of 0.417%, portfolio IH_1 offers the investor a gain in utility of 0.833 percentage points (1.250 – 0.417).

However, as Exhibit 5 indicates, even portfolio IH_1 is suboptimal for the investor with 0.15 residual risk aversion and access to a manager with an IR of 1.0. This investor will maximize utility by holding the original portfolio H_1 (utility of 1.667%). Permitting portfolios beyond the 2% curtain, in this case H_1, provides a gain in utility of 0.417 percentage points (1.667 – 1.250).

SOME IMPLICATIONS

We have raised the 2% curtain to view some of the opportunities that lie beyond it. Not surprisingly, the landscape beyond the curtain abides by the same laws as the landscape within: Greater excess return comes at a cost of greater residual risk. We have found that the slope of the ascent will depend upon the manager's skill, as measured by IR: The higher the IR, the steeper the slope. On any given slope, the optimal portfolio for an investor will depend upon the investor's level of aversion to residual risk. The more regret-

EXHIBIT 4
GAIN IN UTILITY AVAILABLE WITHIN THE 2% CURTAIN

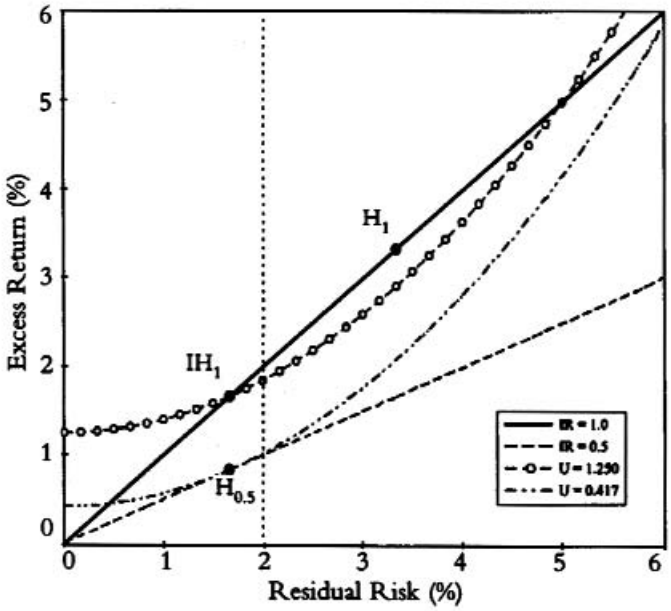
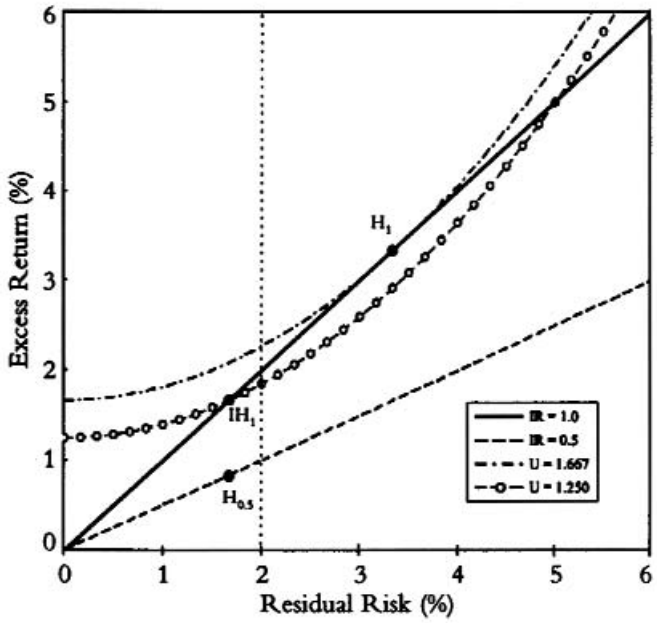


EXHIBIT 5
GAIN IN UTILITY BEYOND THE 2% CURTAIN



averse the investor, the closer to the origin the preferred portfolio will be.

The familiarity of the landscape beyond highlights the artificiality of the curtain itself. Imposition of a constraint such as the 2% limit on residual risk would seem to imply that either excess return (residual risk) drops (rises) precipitously at a given level of residual risk (2% in this case), or that some investors have discontinuous utility functions. These investors would be willing to incur residual risk up to 2% but unwilling even to consider portfolios with residual risks above 2%, whatever their expected returns. Neither of these assumptions seems reasonable.

In fact, imposition of constraints such as the 2% curtain may well encourage suboptimal behavior on the part of investors. Overemphasizing the portfolio's level of residual risk may, as in Exhibit 2, lead investors to sacrifice utility by overestimating their aversion to residual risk. Or it may, as in Exhibit 3, lead them to prefer, in exchange for a low level of residual risk, a less skillful manager.

Constraints on residual risk may also encourage suboptimal behavior on the part of managers. As Grinold [1990, p. 239] has pointed out, there already exist business reasons for high-skill managers to underemploy their insights by taking less than the optimal level of risk:

> Aggressiveness creates a large element of business risk for the manager. Even the most effective active managers will experience significant runs of negative active return with high probability. If they are more aggressive than the other managers employed by the sponsor, they risk being...[last].... Managers with high information ratios should, in general, be more aggressive. However, the high level of aggressiveness may threaten the success of the manager's business. This tension will probably result in less than optimal levels of aggressiveness among skillful managers.

Imposition of risk constraints is likely only to exacerbate this tendency.

This is not to say there are no valid reasons for holding enhanced passive portfolios with residual risk levels below 2%. As we have noted, even at an exceptional manager IR level of 1.0, all investors with residual risk aversions of 0.25 or higher should prefer portfolios with residual risks below 2%. Furthermore, as IRs decrease, optimal residual risk levels for all degrees of residual risk aversion shift downward. Thus, the lower the active manager's level of skill, the lower portfolio residual risk levels should be.

Investors should nevertheless be aware that accepting any arbitrary limit on residual risk may entail a significant sacrifice in utility. They can take two steps to guard against this eventuality. First, they should attempt to determine independently their levels of residual risk tolerance. Low levels of tolerance will lead naturally to portfolios with low residual risk levels; higher levels suggest that higher levels of residual risk, and higher expected excess returns, are more suitable.

Second, investors should actively search out high-IR managers. The higher the manager's IR, the greater the return that can be provided at any given level of risk or any given level of residual risk aversion.

ENDNOTES

[1]The IR is identical to the Sharpe ratio when the latter is measured in terms of excess return and residual risk relative to the underlying benchmark. See Sharpe [1994].

[2]The IR is a linear function of residual risk when short selling is unrestricted and liquidity is unlimited. In practice, the IR slope will decline at high levels of residual risk.

[3]Equation (4) is derived by setting the first derivative of U with respect to ω equal to zero.

[4]The underlying benchmark can be thought of as a risk-free asset in this context, as it is riskless for the investor concerned only with excess return and residual risk.

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